

MATHEMATICS AND WORD PLAY

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Many mathematicians enjoy wordplay, and for obvious reasons. It is almost a branch of combinatorial mathematics. The pleasure derived from solving a combinatorial problem is very much like the pleasure of solving a cryptogram or a crossword puzzle, or constructing a good palindrome. Given the formal system of arithmetic, ancient mathematicians asked themselves whether the digits 1 through 9 could be placed in a three-by-three matrix so that rows, columns, and the two diagonals had the same sum. This is not much different from asking if, given the formal rules of English, one can construct a three-by-three word square in which each row, column, and main diagonal is a different word.

There is, of course, a difference between combinatorial mathematics and wordplay. Mathematics is embodied in the structure of the universe. Although mathematical systems are free inventions of human minds, they have astonishing applications to nature. No one expected non-Euclidian geometry to be useful, but it proved to be essential to relativity theory. Boolean algebra seemed useless until - surprise! - it turned out to model the electrical networks of computers. There are hundreds of other outstanding instances of what physicist Eugene Wigner has called the "unreasonable effectiveness of mathematics".

Think of the letters and words of a language, together with its rules, as a formal system. Although the words have arbitrary meanings assigned to them by minds, and there may be a "deep structure" of syntax that conforms to logic, the words themselves have no reality apart from a culture. Butterflies are all over the world, but you will not find the word "butterfly" by looking through a telescope or microscope. However, once the word becomes attached to butterflies, it is amusing to observe that butterflies flutter by. Because language, unlike mathematics, is "artificial", wordplay has more in common with, say, inventing card tricks or playing chess.

The combinatorial nature of wordplay is underscored by the recent use of computers for solving word problems. Disks containing all the words of a language are now available. With suitable programs they can be used to construct word squares, find anagrams, shortest word ladders, and so on. I wouldn't be surprised if some day computers will solve complicated crosswords as easily as they now solve chess problems.

It is worth noting that both in mathematics and wordplay, solv-

ing a problem is curiously like confirming a theory. In solving a cryptogram, for example, one first makes conjectures. Is a single-letter word A or I, or maybe O? Is ABCA the word "that"? Such conjectures are then tested to see if they lead to contradictions. If they lead to other words, they gain in their probability of being correct. Eventually a point is reached at which one is certain that a cryptogram has been cracked even though not all its letters are known.

One is tempted to say that when all words are known one can be absolutely certain a cryptogram has been solved. This is not the case because, especially if the cryptogram is short, there just could be another solution that the composer of the puzzle intended. If, however, the cryptogram is long, such uncertainty becomes vanishingly small. This is true also in science. When there is a large abundance of facts explained by a theory, such as by the Copernican theory or the theory of evolution, certainty reaches a probability of 0.99999999...

Now for a deep metaphysical question. If chess had not been invented, is there a sense in which theorems about chess can be said to exist? Assuming the formal system of chess, and given a certain position on the board, is it permissible to say that there is a mate in three moves even if no one has posed the problem? Assuming the structure of a deck of cards, is there a sense in which a good card trick is somehow "out there," in a Platonic realm of universals, even if no cards existed?

Suppose there were no English language. Would it be meaningful to say that given such a language, there is a sense in which a certain anagram "exists" even if no one spoke English? It is something like asking if a certain number with a million digits is prime or composite before anyone has tested the number to find out. Well, not quite, because arithmetic certainly "exists" as a formal system. Anyway, most mathematicians are Platonists who believe that, no matter how bizarre, or how far removed from reality a system can be, they "discover" its theorems rather than invent them. Even though English is a human construction, nowhere to be found in nature, is there a sense in which its wordplays are "real" before anyone finds them? I leave answering this to my readers.